How to Prove Twin Prime Conjecture

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In this document, we provide a sketch of how to prove the twin prime conjecture and generally address the infinitely often occurrence of bounded gap between primes. Please go to the next page.

1. INTRODUCTION

We begin with the following theorem.

Theorem 1. *Any prime number p_N can be written as*

$$2+\sum_{\substack{n\leq N\\ p_{n+1}-p_n=1}} 1+2\sum_{\substack{n\leq N\\ p_{n+1}-p_n=2}} 1+4\sum_{\substack{n\leq N\\ p_{n+1}-p_n=4}} 1+\ldots+2k\sum_{\substack{n\leq N\\ p_{n+1}-p_n=2k}} 1=p_N \tag{1}$$

Proof. Let $p_1 < p_2 < p_3 \ldots < p_N$ be the first N primes. We can write

$$p_1+(p_2-p_1)+(p_3-p_2)+\ldots+(p_N-p_{N-1})=p_N \tag{2}$$

where each $p_j - p_{j-1}$ is an even number. One has

$$2+1+2t_1+4t_2+\ldots+2kt_k=p_N \tag{3}$$

where t_1 is the number of the pair of twin primes, t_2 are cousin prime, t_3 are sixy primes etc. The result follows if we write

$$t_k=\sum_{\substack{n\leq N\\ p_{k+1}-p_k=2k}} 1 \tag{4}$$

□

From theorem 1, we have

Theorem 2.

$$3+2\sum_{1\leq k\leq k_{\max}} kt_k=p_N \tag{5}$$

where t_k is given by (4)

If $p_N \to \infty$, then either $k_{\max} \to \infty$ or $t_k \to \infty$. If $k_{\max} \to \infty$, this means it is an infinite sum with each sum is finite. And that each bounded gap occurs finitely often. If $t_k \to \infty$, then the kth bound occurs infinitely often. If $t_1 \to \infty$, this proves the twin prime conjecture