Time, Schrödinger Equation, General Relativity, and Quantum Gravity

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In this paper, we propose a solution of the problem of time in quantum gravity. This is done by first defining what time actually is. We find that time is a collection of frozen moments or instants. Time flows at the cost of creating internal or hidden coordinates. The usual rules of calculus do not hold for time. To allow the flow of time, we have to modify the calculus. We call it the mechanics of evolving constants (MEC) that allows constants to evolve. MEC requires modifying Newtonian dynamics, quantum mechanics, general relativity, and quantum gravity.

1. INTRODUCTION

One of the biggest problems in physics is to unify quantum mechanics (QM) and general relativity (GR). The problem arises due to different notions of time in the two theories. In QM, time is a parameter that is absolute and flows equably everywhere. Whereas in GR, time is dynamical and is a component of four-dimensional spacetime.

In quantum gravity time totally disappears [1–3]. An example is the Wheeler-DeWitt (WdW) equation ($\hat{H}\Psi(q) = 0$), where $\hat{H}$ is the Hamiltonian and Ψ is the wave function of the gravitational field q [4, 5]. The 0 on the right of WdW equation means there is no time. It is possible to keep time in the theory, but then to give up space as in [6].

Physicists have different views on the nature of time. According to Julian Barbour, time is an illusion [7]. Lee Smolin thinks that time is real [8]. As time disappears in quantum gravity, Carlo Rovelli maintains it must be possible to treat time on the same footing as with the other physical variable but not an independent variable [2].

Another aspect of time is the directionality and the flow of time. A great deal of research is also devoted to the study of the flow of time [9–11].

A. Caticha, in [12], [13] sets criteria for something to be called time: (a) something one might identify as an instant, (b) a sense in which these instants can be ordered, (c) a convenient concept of duration measuring the separation between instants.

In this paper, we define time. The expression of time (25) suggests that each instant of time is a frozen moment or instant. It leads to the mechanics of evolving constants (MEC). It is noted that there are two types of constants. One is called the true constants which do not change if time. The second type of constants evolve in time. The second new ingredient is a principle or rule that we identify with the derivative of symbols or letters which works like the product rule of derivatives that involve the coupling of true and evolving constants (equation (37) below). The rule of symbol derivative allows us to modify any fundamental equation in physics such as Newtonian dynamics, Schrödinger equation, Einstein's field equation, and Wheeler-deWitt equation.

2. PROBABILISTIC NATURE OF TIME

We assume that time comes in three forms: past, present, and future. Let Ω be a sample space of the future events that may likely happen

$$\Omega = \{E_1, E_2, \ldots E_n\}, \tag{1}$$

where the subscript n may tend to infinity. As soon as an event say E_1 happens, it is removed from the future and added to the present. Let $P(E_i)$ be the probability of the event E_i. For simplicity we write $P(E_i)$ as P_i. Next, we introduce the function $g(P_1, P_2, \ldots, P_n)$, which we call it the ignorance function. This function is defined by

$$g(P_1, P_2, \ldots, P_n) := \prod_{i=1}^{n} (a_i P_i + b_i). \tag{2}$$

One can note that it is linear in each of its arguments. Here a_i and b_i are constants that are determined by initial conditions.

In order to create an event say E_1, one has to differentiate (2) w.r.t. P_1,

$$g(P_2, P_3, \ldots P_n) = \frac{\partial}{\partial P_1} \prod_{i=1}^{n} (a_i P_i + b_i) = a_1 \prod_{i=2}^{n} (a_i P_i + b_i). \tag{3}$$

By differentiating w.r.t. P_1 does not take us to present fully. To obtain the present one has to differentiate w.r.t. to all the probabilities,

$$g_0 = \prod_{i=1}^{n} \frac{\partial^i}{\partial P_i} (a_i P_i + b_i) = \prod_{i=1}^{n} a_i. \tag{4}$$

Since g_0 is independent of all probabilities, we call it the sureness function.

A. Definition of Time

The purpose of introducing the function g is to define time. Thus time is defined as

$$t[P_i] = -\tau \log g(P_1, P_2, \ldots, P_n) = -\tau \sum_{i=1}^{n} \log(a_i P_i + b_i)\,, \tag{5}$$

where τ is a constant which sets the units of time[14].

We now find the constants a_i and b_i. There are two cases

1. Case 1: When $t[P_i] = 0$, then $P_i = 1$. And when $t[P_i] \to \infty$, then $P_i = 0$. The first part of this case implies that

   $$a_i + b_i = 1\,, \text{ for all } \; i\,. \tag{6}$$

   where we have used $\log 1 = 0$ The second part of case 1 implies

   $$b_i = 0 \text{ for all } \; i\,. \tag{7}$$

   Here we used the fact as $x \to 0_+$, $\log x \to -\infty$ From the last two equations, we get $a_i = 1$ for all i. Therefore

   $$t^+[P_i] = -\tau \sum_i \log P_i \tag{8}$$

   For later convenience, we denoted this time by t^+.

2. Case 2: When $t[P_i] = 0$, then $P_i = 0$. And when $t[P_i] \to \infty$, then $P_i = 1$. The first part of this case implies

   $$b_i = 1\,, \text{ for all } \; i\,. \tag{9}$$

   The second part of case 2 implies

   $$a_1 + b_1 = 0\,, \text{for all} i\,. \tag{10}$$

   This gives $a_i = -1$ for all i. Therefore

   $$t^-[P_i] = -\tau \sum_i \log(1 - P_i)\,. \tag{11}$$

Since time is a continuous variable, the summations in equations (8) and (11) need to be replaced by integration.

$$-\tau \sum_i \log P_i \to -\tau \int dx \log P(x)\,,$$

and

$$-\tau \sum_i \log(1 - P_i) \to -\tau \int dx \log(1 - P(x))\,,$$

because under a change of variable $x \to y = y(x)$, the actual probabilities should not change. We must have [15]

$$P(x)dx = P'(y)dy\,.$$

Therefore, expressions for equations (8) and (11) would be

$$t^+ = -\tau \int dx g^{1/2}(x) \log \frac{P(x)}{\mu(x)}\,, \tag{12}$$

and

$$t^- = -\tau \int dx g^{1/2}(x) \log \left(1 - \frac{P(x)}{\mu(x)}\right)\,, \tag{13}$$

where $g(x) = \det g_{ab}(x)$ is the determinant of the metric $g_{ab}(x)$ and $\mu(x)$ is the prior probability distribution.

The two expressions for time t^+ and t^- need detailed analysis. First, we analyze t^+. Our first observation shows that $\delta t^+ = 0$. This is so because the right hand side of equation (12) is constant as the integration is carrying over x and $P(x)$ only depends on x. This means that t^+ is a frozen moment or an instant, it is not flowing. So how could one obtain a later instant of time? Interestingly, it is possible. Let us first write equation (14) as

$$t_1^+ = -\tau \int dx_a g^{1/2}(x_a) \log \frac{P(x_1)}{\mu(x_a)} \,, \tag{14}$$

Let $t_2^+ = t_1^+ + \delta t^+$ be a later instant of time defined by

$$t_2^+ := -\tau \int g^{1/2}(x_1) g^{1/2}(x_2) dx_a dx_b \log \frac{P(x_a, x_b)}{\mu(x_a, x_b)} \,, \tag{15}$$

where $P(x_1, x_2)$ is the joint probability distribution of x_1 and x_2. On has the interval

$$\delta t^+ = t_2^+ - t_1^+ = -\tau \int dx_1 dx_2 g^{1/2}(x_1) g^{1/2}(x_2) \log \frac{P(x_1, x_2)}{\mu(x_1, x_2)} + \tau \int dx_1 g^{1/2}(x_1) \log \frac{P(x_1)}{\mu(x_1)} \,, \tag{16}$$

where x_1 is contained in the sample space $\mathcal{X}_1$. We can also call it the dimension. Whereas, x_2 is an element of the dimension $\mathcal{X}_2$. It means that time advances at the price of spontaneously creating dimensions. We note that x_1 and x_2 do not appear in the final result. This means that $\mathcal{X}$'s are internal dimensions.

Next we show that $t_1^+ \leq t_2^+$. Since $P(x_1, x_2)$ is joint probability. Using the product rule of probability one has

$$P(x_1, x_2) = P(x_2|x_1) P(x_1) \,. \tag{17}$$

Furthermore, probability distributions take values between 0 and 1. We have

$$\begin{aligned} 0 \quad &\leq P(x_2|x_1) \leq \quad 1 \\ 0 \ &\leq P(x_2|x_1)P(x_1) \leq \ P(x_1) \\ 0 \quad &\leq P(x_1, x_2) \leq \quad P(x_1) \end{aligned} \tag{18}$$

One has

$$\log P(x_1, x_2) \leq \log P(x_1) \,, \tag{19}$$

This implies

$$-\tau \int g^{1/2}(x_2) g^{1/2} dx_a dx_2 \log \frac{P(x_1, x_2)}{\mu(x_1, x_2)} \geq -\tau \int g^{1/2}(x_1) g^{1/2}(x_2) dx_1 dx_2 \log \frac{P(x_1)}{\mu(x_1)} = -\tau V_2 \int dx_1 \log P(x_1) \tag{20}$$

where V_2 is a volume. This gives

$$t_2^+ \geq V_2 t_1^+ \,. \tag{21}$$

It will be shown later in the case of Newtonian time, $V_b \geq 1$. This gives $t_a^+ \leq t_b^+$. This means t^+ flows forward.

In a similar way, one can show $t_1^- \geq t_2^-$. This means that t^- flows backward. Here

$$t_1^- = -\tau \int g^{1/2} dx_1 \log \left(1 - \frac{P(x_1)}{\mu(x_1)}\right) \,, \tag{22}$$

and

$$t_2^- = -\tau \int g^{1/2}(x_1) g^{1/2}(x_2) dx_1 dx_2 \log \left(1 - \frac{P(x_1, x_2)}{\mu(x_1, x_2)}\right) \,. \tag{23}$$

$$\delta t^- = t_b - t_a = -\tau \int g^{1/2}(x_a) g^{1/2}(x_b) dx_a dx_b \log \left(1 - \frac{P(x_a, x_b)}{\mu(x_a, x_b}\right) + \tau \int g^{1/2}(x_a) dx_a \log \left(1 - \frac{P(x_a)}{\mu(x_a}\right) \,, \tag{24}$$

One can observe that $\delta t^- \leq 0$.

3. DEFINITION OF TIME

We define an instant of time as follows

$$T_n = \int_{a_1}^{b_1} \ldots \int_{a_n}^{b_n} dx_1 \ldots dx_n f(x_1, \ldots x_n) \;\; n \in \mathbb{N} \tag{25}$$

where f_n is a function of n arguments which can be chosen for a given system. One can see the right side of (25) involves definite integrals. When f_n is chosen and the integration is performed, one gets a constant. This means that T_n is a frozen moment or an instant. Here the arguments x's can be thought of as internal coordinates or a data set that does not appear in the final result.

The next instant can be defined as follows

$$T_{n+1} = \int_{a_1}^{b_1} \ldots \int_{a_{n+1}}^{b_{n+1}} dx_1 \ldots dx_{n+1} f(x_1, \ldots x_{n+1}) \, . \; n \in \mathbb{N} \tag{26}$$

One can note that it contains $n + 1$ argument. It means that time flows at the cost of creating internal coordinates.

The interval of time can be defined as

$$\Delta T_n = T_{n+1} - T_n \, , \tag{27}$$

where T_n and T_{n+1} are given by (25) and (26) respectively. The interval ΔT_n can be positive, negative, or zero depending on the choice of f_n. As such there is no restriction on f_n. However, if f_n has a property P, so as f_{n+1}. For example, if f_n is Gaussian in n variables, then f_{n+1} is Gaussian in $n + 1$ variables.

Next we consider a few functions as examples to find the instant of time and interval of time.

Example 1. Let $f_n = \sum_{k=1}^{n} x_n$ be defined on the interval $[0, 1] \times \ldots [0, 1]$. Then

$$T_1 = \int_0^1 dx_1 x_1 = \left. \frac{x^2}{2} \right|_0^1 = 1/2. \tag{28}$$

It is not difficult to show that

$$T_n = \frac{n}{2} \, , \quad T_{n+1} = \frac{n+1}{2} \, , \text{ and } \Delta T_n = 1/2 > 0 \, . \tag{29}$$

This can be thought of as Newtonian time where time flows equably as the interval of time is constant.

Example 2. Consider

$$T_n = \int_0^1 \ldots \int_0^1 dx_1 \ldots dx_n \prod_{k=1}^{n} 2^n x_k \, . \tag{30}$$

One obtains $T_n = 1$ for all n. Since T_n is constant, this means that $\Delta T_n = 0$. This may correspond to a photon in its rest frame where the time interval is zero.

Example 3. Suppose

$$T_n = \int_0^1 \ldots \int_0^1 dx_1 \ldots dx_n \prod_{k=1}^{n} x_k = \frac{1}{2^n} \, . \tag{31}$$

One has $\Delta T_n = -1/2^{n+1}$. Since this interval is negative, it means time flows backward in this case.

4. THE CALCULUS OF CONSTANTS

As we noted above that each instant of time is a constant, which means any function of time would also be a constant. Furthermore, the derivative of a constant function is zero. We define the derivative of a constant function as follows.

Definition 1. Let $G(T)$ be function of T, the derivative is defined as follows

$$\frac{\Delta G(T)}{\Delta T_n} = \frac{G(T_{n+1}) - G(T_n)}{T_{n+1} - T_n}\,. \tag{32}$$

One can observe that it simply involves the division of numerator over denominator. We wish to explain with an example.

Example 4. Consider the function in example 1. Let $G(T) = T^3$. Find the derivative of G.
We have $T_n = n/2$ and $\Delta T_n = 1/2$. One has

$$\frac{\Delta G}{\Delta T_n} = \frac{((n+1)/2)^3 - (n/2)^3}{1/2} = \frac{(n+1)^3 - n^3}{4} \tag{33}$$

5. TRUE CONSTANTS VS EVOLVING CONSTANTS

We observe that there are two types of constants. One are those that stay constant. An example is given by example 2 and other constants are those that vary. We will denote the true constants by lowercase letters $a, b, c\ldots$ and the evolving constants will be denoted by uppercase letters $A, B, C, \ldots$. The lowercase letters satisfy all the properties of ring theory or field theory such as $a + b = b + a$, $a + 0 = 0 + a = a$, $a + (-a) = 0$ etc. But the uppercase letters do not have these properties. When we write numbers such as integers, the true constants would simply be written as $0, 1, 2, 3, \ldots$. On the other hand, evolving constants would carry indices $0_{f_n}, 1_{f_n}, 2_{f_n}, 3_{f_n}, \ldots$. The evolving integers oscillate about their true values. In one example we can calculate 0_{f_n}

$$0_{f_n} = \int_0^1 \ldots \int_0^1 dx_1 \ldots dx_n \sum_{k=1}^{n} (-1)^{k+1} x_k\,. \tag{34}$$

This gives

$$0_{f_n} = \begin{cases} 0 & \text{when } n \text{ is even} \\ 1/2 & \text{when } n \text{ is odd} \end{cases} \tag{35}$$

There is some ambiguity in the oscillating numbers. One may also have

$$(1/2)_{f_n} = \begin{cases} 0 & \text{when } n \text{ is even} \\ 1/2 & \text{when } n \text{ is odd} \end{cases} \tag{36}$$

This ambiguity is understandable as oscillatory numbers are not unique. In the same way, one has $1_{f_n} = 0_{f_n} + 0_{f_n}$, $(3/2)_{f_n} = 0_{f_n} + 0_{f_n} + 0_{f_n}$, $2_{f_n} = 0_{f_n} + 0_{f_n} + 0_{f_n} + 0_{f_n}$ etc. This also suggests that there is intrinsic oscillation or fluctuation in time.

Next, we define a very important concept, we call it the derivative of letters or symbols which works exactly like the product rule of derivatives. According to the product rule, one differentiates one variable and treats the second variable constant and then treats the first variable constant and differentiates the second one. Here for clarity, we consider three uppercase A, B, and C as follows

$$ABC \to aBC + AbC + ABc\,. \tag{37}$$

Here, for instance, in the first term on the right A is lowered whereas B and C remain uppercase. Sometimes we may also use

$$ABC \to A_{f_n}BC + AB_{f_n}C + ABC_{f_n}\,. \tag{38}$$

The latter notation is used for two reasons. First, as we noted in the examples above different internal functions, f_n, return different results, It is acceptable as it corresponds to different solutions of a given equation. Secondly, this notation is convenient when dealing with integers or constants of nature such as

$$6 = 2 \cdot 3 \to 2_{f_n} \cdot 3 + 2 \cdot 3_{f_n},. \tag{39}$$

We are now in the position to find commutators. Consider

$$\begin{aligned} AB &\to aB + Ab \\ BA &\to bA + Ba\,. \end{aligned} \tag{40}$$

On combining one gets the desired commutators

$$[A, B] = [a, B] + [A, b] \tag{41}$$

Note that here the various commutators do not involve any kind of operators as one would expect in QM. Therefore each commutator should be an oscillatory zeros. Therefore

$$[A, B] = 0_{f_n}\,, [a, B] = 0_{f_n}\,,\ \text{and}\ [A, b] = 0_{f_n}\,. \tag{42}$$

Note that lowercase letters commute such as $[a, b] = 0$, where 0 is the true zero.

6. NEWTONIAN DYNAMICS

Let $X^i(T_n)$ be the initial state of the particle, where $i = 1, 2, 3$ are spatial coordinates. Let $X^i(T_{n+1})$ be the later state. The velocity is defined as

$$V^i(T_n) = \frac{X^i(n+1) - X^i(n)}{T_{n+1} - T_n}\,. \tag{43}$$

Note that equation (43) describes time evolution. The particle does not move in space. For if then the particle position would change from X^a to X'^i. One can see that the prime coordinates nowhere appear in equation (43). If there is no motion in space then what does (43) describe? We interpret that V^i as the velocity of aging. The particle may undergo a decaying process. In other words, the particle is weathered over time. One must also note that V^i can be positive, negative, or zero. It means that an experiment can be devised to stop aging.

Next, we define momentum as follows

$$P^i = MV^i\,. \tag{44}$$

Note that whenever we recall an equation, it must be written in uppercase letters and then apply the derivative of the symbols to generalize it. The momentum generalizes as follows

$$P^i = MV^i \to mV^i + Mv^i\,, \tag{45}$$

where m and v^i are the true constant mass and true constant velocity respectively.

Similarly, recall Newton's second law of motion and revise it

$$F^i = MA^i \to mA^i + Ma^i\,, \tag{46}$$

where A^i and a^i are evolving and true accelerations. Whereas m and M are the true and evolving masses. If one interprets m to be the inertial mass and M is the gravitational mass and A^i is the acceleration of m and a^i is the acceleration due to gravity, one observes that m and M are not equal. This violates the weak equivalence principle (WEP) according to which inertial and gravitational masses of a body are equal [16]. We interpret that A^i and M are regulators. That is A^i oscillates in such a way to keep m constant. Whereas M regulates a^i constant.

7. NON-RELATIVISTIC QUANTUM MECHANICS

Note that MEC does not challenge any branch of physics such as classical mechanics (CM) or quantum mechanics (QM). Rather it revises it. Just like when one goes from CM to QM, one replaces physical quantities with their operator counterparts. Here we use the derivative of symbols to revise any equation.

We begin with the commutator

$$[\hat{x}, \hat{p}] = i\hbar\,, \tag{47}$$

where $\hat{x}$ and $\hat{p}$ are the position and momentum operators, $i = \sqrt{-1}$, and $\hbar$ is the Planck's constant. One notes that MEC does not involve operators, rather it involves uppercase and lowercase letters. Let us first fix the left side of equation (47)

$$[\hat{x}, \hat{p}] \to [X, P] \to [x, P] + [X, p]\,, \tag{48}$$

Also, revise the right side of (47)

$$i\hbar \to i\hbar_{f_n} + i_{f_n}\hbar\,, \tag{49}$$

where i_{f_n} and $\hbar_{f_n}$ are the evolving counterparts of i and $\hbar$. When equations (48) and (49) are combined, one has

$$[x, P] = i\hbar_{f_n}\,, \text{ or } [x, P] = i_{f_n}\hbar\,. \tag{50}$$

Similarly,

$$[X, p] = i_{f_n}\hbar\,, \text{ or } [X, p] = i\hbar_{f_n}\,. \tag{51}$$

The uncertainty principle also needs to be revised. In QM, the uncertainty principle is given by

$$\Delta x \Delta p \geq \frac{1}{2}\hbar\,. \tag{52}$$

Now revise it

$$\Delta x \Delta p \to \Delta X \Delta P \to \Delta X \Delta p + \Delta x \Delta P \geq \left(\frac{1}{2}\right)_{f_n} \hbar + \frac{1}{2}\hbar_{f_n}\,. \tag{53}$$

Here $\Delta X = X(T_{n+1}) - X(T_n)$ is the change in the position of the particle in time. Similarly, $\Delta P = P(T_{n+1}) - P(T_n)$. Interestingly, $\Delta x = 0$ and $\Delta p = 0$. This is because lowercase symbols do not change in time. It yields

$$\Delta X \Delta P = 0_{f_n}\,, \tag{54}$$

Since 0_{f_n} oscillates, it means that there is a time when position and momentum can be measured exactly.

Next, we recall the Schrödinger equation in the following form

$$i\hbar \frac{\Delta \Psi_n}{\Delta T_n} = \hat{H}\Psi, . \tag{55}$$

We want to revise it by taking derivatives of symbols. There is a technical issue with the derivative of Ψ on the left. Upon revising the difference leads to $\frac{0}{0}$ form. To avoid we proceed as follow.

$$i\hbar \left(\frac{1}{\Delta T_n}\right) \Delta \Psi_n = \hat{H}\Psi\,. \tag{56}$$

Now revise it

$$i\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right) (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right)_{f_n} \Delta \psi_n = H\psi + h\Psi \tag{57}$$

The last term on the left is zero as it involves the difference of lowercase ψ. One has

$$ii\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right) (\Delta \Psi_n)_{f_n} = H\psi + h\Psi\,, \tag{58}$$

where on the right, H is the classical Hamiltonian

$$H = \frac{P^2}{2M} + \Phi\,. \tag{59}$$

where Φ is scalar potential and h can be thought of as intrinsic Hamiltonian. One may have

$$h = \mu_m \vec{\sigma} \cdot \vec{b}\,, \tag{60}$$

where μ_m is the magnetic moment of the particle, $\vec{\sigma}$ are Pauli matrices, and $\vec{b}$ is the magnetic field that does not vary in time. The Pauli spin matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,, \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{61}$$

Thus the Schrödinger equation (58) becomes

$$i\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar \left(\frac{1}{\Delta T_n}\right)_{f_n} (\Delta \Psi_n)_{f_n} + i_{f_n}\hbar_{f_n} \left(\frac{1}{\Delta T_n}\right) (\Delta \Psi_n)_{f_n} = H\psi + \mu_m \vec{\sigma} \cdot \vec{b}\Psi\,, \tag{62}$$

which can be thought of as Schrödinger-Pauli equation of spin-1/2 particles.

8. SOLUTION OF SCHRÖDINGER EQUATION (SE)

In order to solve SE in QM, one must first specify the potential Φ and the boundary conditions. One may also use various approximation methods or perturbation theory to solve it. On the other hand, in MEC, it is possible to exactly solve SE for any potential. First, there is the space derivative that enters through the Laplacian on the right. In MEC, only time derivatives exist. Again, in MEC, there are no such derivatives. We note the derivative introduced in section 4 involves the division of the numerator over the denominator.

We want to solve equation (62). One can observe that on the left i_{f_n}, h_{f_n}, and $(1/\Delta T_n)_{f_n}$ are oscillatory that depends on f_n. Luckily, we have freedom in choosing f_n such that

$$i_{f_n} = 0_{f_n}\ ,\ h_{f_n} = 0_{f_n},\ \text{and}\ \left(\frac{1}{\Delta T}\right)_{f_n} = 0_{f_n}\ , \tag{63}$$

where the various 0_{f_n} in equation (63) may not be the same. Now use equation (63) in (62), one has

$$0_{f_n} + 0_{f_n} + 0_{f_n} = H\psi + \mu_m \vec{\sigma}\cdot\vec{b}\Psi\ , \tag{64}$$

Since 0_{f_n} depends on the choice of f_n, thus a family of solutions can be obtained for various f_n.

9. GENERAL RELATIVITY

According to MEC, space does exist. However, variation only happens in time but not in space. So we will only keep time derivatives in what follows. Let us remind ourselves Einstein's equations [16]. One has

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}\ , \tag{65}$$

where G is the Newtonian gravitational constant, $G_{\mu\nu}$ is Einstein tensor, and $T_{\mu\nu}$ is energy-momentum tensor. Apply MEC to (65) to revise it

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \rightarrow 8\pi_{f_n} G_{f_n}(T_{\mu\nu})_{f_n} + 8_{f_n}\pi G_{f_n}(T_{\mu\nu})_{f_n} + 8_{f_n}\pi_{f_n} G(T_{\mu\nu})_{f_n} + 8_{f_n}\pi_{f_n} G_{f_n} T_{\mu\nu}\ , \tag{66}$$

The Einstein tensor is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\ , \tag{67}$$

where the Ricci tensor is

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}\ , \tag{68}$$

and the Ricci scalar is

$$R = g^{\mu\nu} R_{\mu\nu}\ . \tag{69}$$

Furthermore,

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}\ , \tag{70}$$

where

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu})\ , \tag{71}$$

are connection coefficients aka Christoffel symbols. In order to solve Einstein's equation, we go in reverse order. According to MEC, there is no variation in space, things happen in time. So we only keep time derivatives.

$$\Gamma^{0}_{00} = \frac{1}{2} g^{00}\left(\frac{\Delta g_{00}}{\Delta T} + \frac{\Delta g_{00}}{\Delta T} - \frac{\Delta g_{00}}{\Delta T}\right) = \frac{1}{2} g^{00}\frac{\Delta g_{00}}{\Delta T} \tag{72}$$

Now revise it

$$\gamma^{0}_{00} = \left(\frac{1}{2}\right)_{f_n} g^{00}\left(\frac{1}{\Delta T}\right)\Delta g_{00} + \frac{1}{2} g^{00}_{f_n}\left(\frac{1}{\Delta T}\right)\Delta g_{00} + \frac{1}{2} g^{00}\left(\frac{1}{\Delta T}\right)_{f_n}\Delta g_{00} + \frac{1}{2} g^{00}\left(\frac{1}{\Delta T}\right)(\Delta g_{00})_{f_n}\ . \tag{73}$$

We can assign 0_{f_n} to each term on the right and then absorb all the 0_{f_n} in a single one. One then has

$$\gamma^0_{00} = 0_{f_n} \,. \tag{74}$$

The same goes for Ricci scalar and tensor. It means

$$R_{00} = 0_{f_n} \,, \text{ and } R = 0_{f_n} \tag{75}$$

The Einstein tensor becomes

$$G_{00} = 0_{f_n} \tag{76}$$

Finally, equation (66) yields

$$0_{f_n} = 0_{f_n} + 0_{f_n} + 0_{f_n} + 8\pi G t_{00} \,, \tag{77}$$

which gives after rearranging

$$t_{00} = \frac{0_{f_n}}{8\pi G} \,. \tag{78}$$

10. QUANTUM GRAVITY

In this section, we briefly discuss the Wheel-deWitt equation [4]

$$\left[(q_{ab}q_{cd} - \frac{1}{2}q_{ac}q_{bd}) \frac{\delta}{\delta q_{ac}} \frac{\delta}{\delta q_{bd}} - \det q R[q] \right] \Psi(q) = 0 \,, \tag{79}$$

where q_{ab} is spatial metric with $a, b = 1, 2, 3$. $R[q]$ is the Ricci scalar, and $\det q$ is the determinant of q_{ab}.

Now apply MEC, first drop the first term on the left as it involves space derivatives, we are left with

$$\det q R[q] \Psi(q) = 0 \,, \tag{80}$$

Now revise it by applying the letter derivative

$$(\det q) R[q]_{f_n} \Psi(q)_{f_n} + \det q_{f_n}(R[q]) \Psi(q)_{f_n} + \det q_{f_n} R[q]_{f_n}(\Psi(q)) = 0_{f_n} \,, \tag{81}$$

where 0_{f_n} on the right is evolving or fluctuating zero.

11. CONCLUSIONS

In this paper, we defined time. We noted that it lead to the mechanics of evolving constants (MEC). We applied MEC, to various branches of physics from Newtonian dynamics, quantum, general relativity, and quantum gravity. The theory of time presented here works for the discrete case. A continuous case will be discussed elsewhere.

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